WATER RESOURCES RESEARCH, VOL. 18, NO. 4, PAGES 1110–1116, AUGUST 1982

Bayesian Analysis of Underground Flooding

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An event-based stochastic model is used to describe the spatial phenomenon of water inrush into underground works located under a karstic aquifer, and a Bayesian analysis is performed because of high parameter uncertainty. The random variables of the model are inrush yield per event, distance between events, number of events per unit underground space, maximum yield, and total yield over mine lifetime. Physically based hypotheses on the types of distributions are made and reinforced by observations. High parameter uncertainty stems from the random characteristics of karstic limestone and the limited amount of observation data. Thus, during the design stage, only indirect data such as regional information and geological analogies are available; updating of this information should then be done as the construction progresses and inrush events are observed and recorded. A Bayes simulation algorithm is developed and applied to estimate the probability distributions of inrush event characteristics used in the design of water control facilities in underground mining. A real-life example in the Transdanubian region of Hungary is used to illustrate the methodology.

The purpose of this paper is to describe the spatial phenomenon of karstic flow into an underground space by means of a Bayesian stochastic model. The construction and operàtion of underground works such as mines, tunnels and subways is strongly affected by possible inflow or inrush events from neighboring aquifers. The characteristics of these future events should be forecasted during the planning process in order that a proper control method can be found. In a karstic aquifer, a stochastic analysis of these characteristics should be performed because of the randomness of fractures and faults in limestone [*Snow*, 1970; *Bogardi et al.*, 1980]. Also, information available during the design phase is generally limited to exploration borings, regional estimation, geological analogy, or subjective judgment. As a result, statistical parameters of the forecasting models are subject to sample uncertainty. After the construction has begun, inrush events are observed, yielding statistical information which should be used to update or improve the forecasts.

In the next section, the forecasting problem is defined and the model is specified. Next, a Bayesian model of the spatial event-based inrush phenomena is constructed and implemented numerically. Model assumptions and possible extensions are discussed. Mine water inrush forecasting in the Transdanubian region of Hungary is used throughout the study to illustrate the procedure.

PROBLEM DESCRIPTION

The information flow related to underground works, such as tunnels, is illustrated in Figure 1. The lifetime design of such works is based on the partial information available during this design phase. Generally, no underground spaces have been opened at the site; consequently, no direct or local observation data on 'geological impact' are available. The phrase 'geological impact' refers to water or sediment inflow from an adjacent aquifer, rock movement caused by opening an underground space, gas inflow, etc. In this paper, water inflows are considered as spatial events, occurring if the underground space meets a fissure or fault in the rock. Thus, as the construction of underground engineering works or the operation of a mine progresses, open spaces grow which causes, in general, more and more inrush events to occur.

Figure 2 shows a typical case of underground works built under karstic water hazard. A rectangular space of given width, height, and horizontal area A has been opened at a depth s under the karstic water level; inrush events that may occur along this area A possess five characteristics that are necessary for designing and operating groundwater control works: (1) the yield $\mathbf{q}$, which can be considered as a steady state flow provided that the total potential energy of the aquifer is large; (2) the distance $\mathbf{L}$ between events projected on the central axis of area A; (3) the number $\mathbf{N}$ of events over A; (4) the maximum yield $\mathbf{q}_{max}$ of inrush events over area A; and (5) the sum $\mathbf{Q}$ of yields over area A.

The design of the control system depends mostly on $\mathbf{Q}$ and $\mathbf{q}_{max}$ [*Kesseru et al.*, 1978; *Duckstein et al.*, 1981]. $\mathbf{Q}$ determines the average capacity of pumps, $\mathbf{q}_{max}$ determines the peak and standby capacities.

It is common practice in mining and underground engineering to characterize the forecast by a single number without providing an error estimate. In a fairly homogeneous porous media subject to small natural uncertainty, such a single-number forecast may be satisfactory. However, expe-

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Paper number 2W0284.
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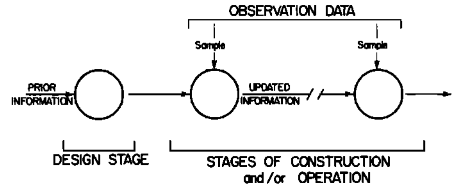

Fig. 1. Information during design and operation stages of underground works.

rience has shown that a number of mines and underground works have been flooded in spite of substantial protection measures taken against such floodings on the basis of a deterministic forecast [*Schmieder,* 1978]. Conversely, there are cases of overdesign in which the control system has not been activated at all during the project lifetime. If, on the other hand, the probability density function (pdf) or at least the mean and variance of the forecast are estimated, then a proper design probability or an economic optimum can be selected. For example, the design of emergency rescue routes is usually based on some small exceedance probability of inrush yield, while a pumping station or grouting scheme may be designed on the basis of minimum expected cost [*Kesseru et al.,* 1978].

In the case study, the stochastic forecasting model is based on the following hypotheses, whose plausibility is justified in the application section.

1. The $\mathbf{q}$ follows a lognormal distribution as shown in Figure 3 for a typical mine.

2. The number of inrush events occurring over an area A, $\mathbf{N}(A)$, follows a Poisson distribution with mean λA, where λ is the specific number of inrushes, as illustrated in Figure 4.

3. The distribution function (DF) of $\mathbf{q}_{max}$ follows its asymptotic maximal distribution [*Feller,* 1968]

$$P(\mathbf{q}_{max} \leq x) = F_{\mathbf{q}_{max}}(x) = \exp[-\lambda A(1 - F_{\mathbf{q}}(x)] \qquad (1)$$

As a consequence, the total yield for area A is the sum of inrush event yields and is calculated as the sum of a Poisson number $\mathbf{N}$ of lognormal inrushes $\mathbf{q}$:

$$\mathbf{Q}(A) = \sum_{i=1}^{\mathbf{N}(A)} \mathbf{q}(i) \qquad (2)$$

The DF of $\mathbf{Q}(A)$ must be determined from the DF of $\mathbf{q}$ and $\mathbf{N}$, unless direct observation data on $\mathbf{Q}(A)$ are available, which is rarely the case.

Three statistical parameters characterize the forecasting model: the mean μ, variance σ^2 of inrush event yields $\mathbf{q}$, and the specific number λ of inrushes. If the values of these

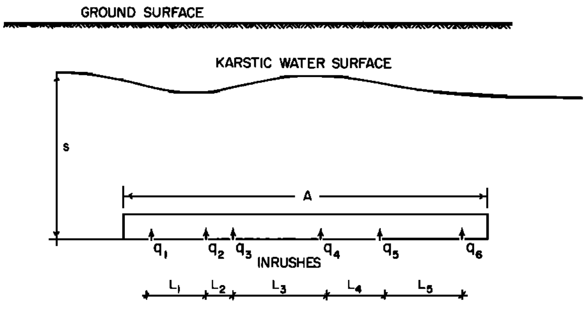

Fig. 2. Inrushes into an underground space.

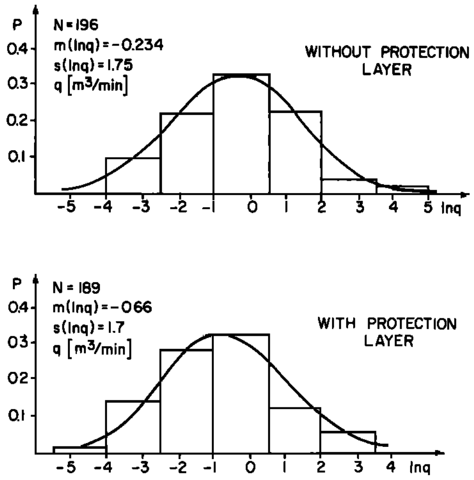

Fig. 3. Distribution of inrush event yield $\mathbf{q}$ from the Dorog mining region.

parameters were known, a forecast fully accounting for natural uncertainty could be made. Again, practically, this is rarely the case. In particular, during the design stage, no on-site information is available. The inrush forecast is thus subject to parameter uncertainty, which is reduced as mining progresses and further observation data are incorporated into updated forecasts. A Bayesian approach provides a means both to account for sample uncertainty and to update forecasts. Furthermore, such an approach provides the elements for economically based decision analysis [*Raiffa and Schlaifer,* 1961].

APPLICATIONS OF THE BAYES MODEL

Features of Bayes decision theory of interest in the present study are well known and may be found, for example, in *Raiffa and Schlaifer* [1961], *Benjamin and Cornell* [1970], *de Groot* [1970], *Davis et al.* [1972], *Wood* [1978], and *Berger* [1980]. Thus we proceed directly to the estimation of the Bayesian distributions of the five random variables defined previously.

Magnitude of Inrush Events q

Since $\mathbf{z} = \ln \mathbf{q}$ is hypothesized to follow a normal distribution with unknown parameters (μ, σ), the joint conjugate

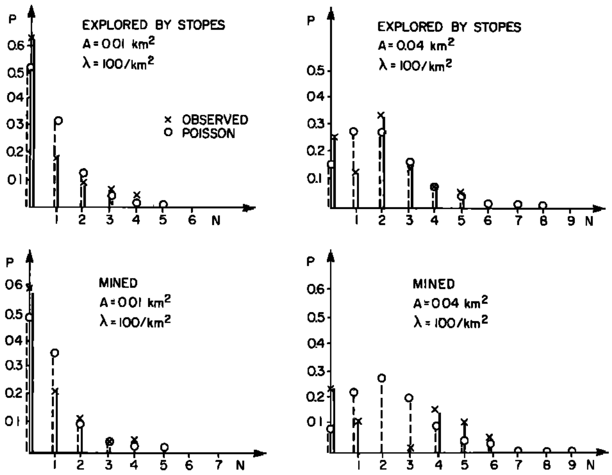

Fig. 4. Distribution of inrush event number $\mathbf{N}$ from the Dorog mining region.

TABLE 1. Data for Prior Estimation of Inrush Yield Characteristics

Coal field	Shafts	A, km^2	FL, km	FL/A, 1/km	Σq, m^3/min	$\Sigma q/A$, m^3/min/km^2	N/A, 1/km^2
Dorog	VI	0.507	6.540	13.0	83.06	161	138
	VIII	0.333	1.865	5.0	18.25	55	10
	X	0.382	1.165	3.0	12.58	34	11
	XII	0.482	3.730	8.0	40.28	83	110
	XV	0.133	2.800	21.0	56.90	420	136
	XVII	0.226	4.715	21.0	75.40	333	115
	Ebszony, Eb	0.391	4.745	12.0	60.42	154	75
	Tokod-Altaro, TA	0.870	20.380	23.4	216.10	248	98
	Tomedek, TOM	0.341	1.570	4.6	16.20	42	62
Tatabanya	III	0.113	3.545	33.0	22.42	200	126
	VI	0.865	8.980	10.4	28.40	33	75
	VII	0.040	1.550	39.0	8.20	208	75
	VIII	0.086	1.300	15.0	1.55	19	19
	X	0.320	3.500	11.0	5.88	18	32
	XI	0.590	4.670	7.9	14.70	25	19
	XII	0.121	0.920	7.6	...	...	...
	XIIa	0.092	1.590	17.0	...	...	...
	XIV	0.526	7.030	13.4	34.42	65	78
	XVb	0.146	0.532	3.7	0.70	5	64
	XVc	0.212	1.730	8.2	...	...	...
	Sikvolgy, S	0.507	7.690	15.3	51.24	100	88

distribution of (μ, σ) is a normal-gamma distribution [*Raiffa and Schlaifer,* 1961; *Zellner,* 1971]. This conjugate distribution has four parameters: the mean and variance of μ and the mean and variance of σ, only three of which are independent. A distinction is made between prior estimation in the design stage and posterior estimation in the construction and operation stages.

Design stage: Prior estimation. No observation data on **q** are available but prior information on the above parameters may be obtained by (1) regional estimation, (2) geological analogy, and (3) use of a hydraulic model and literature data.

1. Regional estimation can be undertaken on the basis of observations in existing mines. As an example, data from two coal fields, Dorog and Tatabanya in the Transdanubian region, Hungary, are given in Table 1, on the basis of the work of *Willems* [1973], who derived a stochastic relationship between the quantity FL/A (FL = total length of faults) and the distance y of the site from the edge of the karstic basin (Figure 5). Prior estimate of the mean of the logarithm of average inrush yield can be given as

$$m' = \frac{\Sigma \bar{z}}{n'} \tag{3}$$

where $\bar{z} = \overline{\ln \mathbf{q}}$ is the average of inrush yield logarithms for a mine shaft, and n' is the number of mineshafts used for the regional analysis; for the example of the Dorog coal field, n'

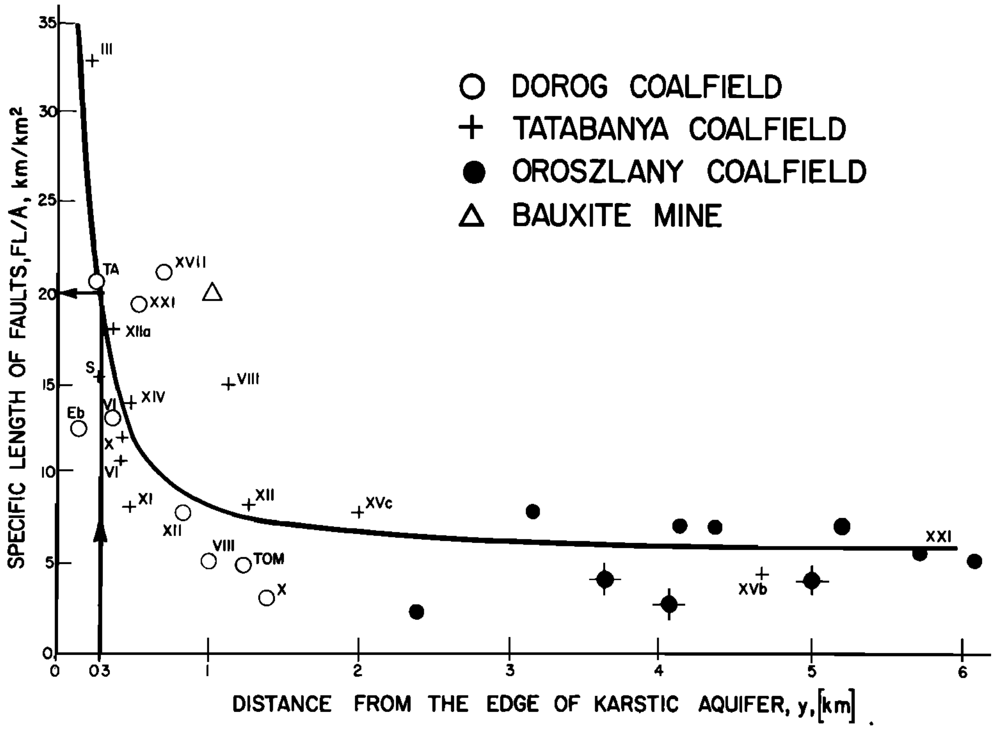

Fig. 5. Relationship between the distance y from the edge of the karstic basin and FL/A [after *Willems,* 1973].

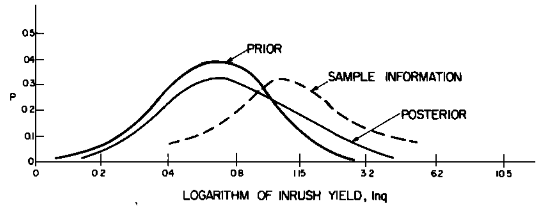

Fig. 6. Simulated prior and posterior distributions of inrush event yield **q**.

= 9. The variance $s^2(\bar{z})$ of the values of $\bar{z}$ can also be estimated. The conjugate prior distribution of μ and σ can be written, after *Benjamin and Cornell* [1970, pp. 628],

$$f'_{\mu,\sigma}(\mu, \sigma) = \left\{ \frac{1}{(2\pi)^{1/2}\sigma/(n')^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{\mu - m'}{\sigma/(n')^{1/2}}\right)^2 \right] \right\}$$
$$\times \left[\frac{((n'-1)/2)^{(n'-2)/2}}{\Gamma((n'-2)/2)} \frac{2}{s'} \left(\frac{s'^2}{\sigma^2}\right)^{(n'-1)/2} \exp\left(-\frac{n'-1}{2} \frac{s'^2}{\sigma^2} \right) \right] \tag{4}$$

Here the three parameters are r', m' and $s'^2 = s'^2(\bar{z})\, n'(n' - 2)/(n' - 1)$.

2. Geological analogy. Often there are not enough regional data to construct regional estimates, but still related information may be available at a few sites in the same geological unit. Let the estimated parameter vector at location y be $\hat{\theta}_j = [m_j, s_j^2(\bar{z})]$, $j = 1, 2, \cdots, J$, and let the probabilities that $\hat{\theta}_j$ is the true parameter vector be p_j; then expected prior parameters can be calculated as

$$m' = \sum_j p_j m_j \tag{5}$$

$$s'^2(\bar{z}) = \sum_j p_j s_j^2(\bar{z}) \tag{6}$$

Probabilities p_j, $j = 1, \cdots, J$ can be estimated subjectively by using the geological analogy between corresponding locations.

3. Hydraulic model and literature data. A hydraulic model for inrush has been developed by *Schmieder et al.* [1975], although the complexity of flow phenomena (laminar and turbulent losses) results in considerable model uncertainty. Nevertheless, an initial guess on inrush parameters can be found from hydraulic calculations and also literature data of inrushes under similar geological conditions.

It may be noted that the use of an improper prior can introduce bias into parameter estimation. Procedure (3) above is more prone to creating bias than (1) or (2), because no averaging or compensation effect can be expected when either a model or published data are used.

Construction and operation stage: Forecast updating. In a given stage of construction or operation, a number n of inrush events has been observed with mean m and variance s^2. Since a conjugate prior distribution is used, the posterior distribution has parameters n'', m'', s''^2, calculated from prior and observed parameters as shown by *Benjamin and Cornell* [1970]

$$n'' = n + n' \tag{7}$$

$$n'' m'' = nm + n'm' \tag{8}$$

$$(n'' - 1)s''^2 + n'' m''^2 = [(n - 1)s^2 + nm^2] + [(n' - 1)s'^2 + n'm'^2] \tag{9}$$

The prior distribution of **N** may be determined from regional considerations; in the absence of any information, a noninformative prior may be used [*Jeffreys*, 1961; *Jaynes*, 1968; *Box and Tiao*, 1973; *Villegas*, 1977; *Bernardo*, 1979; *Berger*, 1980].

Bayesian distributions of **q** can be estimated by the following simulation algorithm:

1. Generate a pair (μ, σ) by using the normal-gamma distributions (equation (4)) first with prior parameters, then with posterior parameters.
2. Generate a normal variate α_j by using parameters (μ, σ) obtained in step 1.
3. Store the prior or posterior Bayesian value of q, $q_j = \exp(\alpha_j)$, then go back to step 1.

These values of q_j can be used in subroutines to derive distributions of random variables which do not have closed form Bayesian distributions or else to perform decision analyses.

As an example, consider a mine to be located in the Tatabanya coal field. By using data shown in Table 1, the three parameters of the prior conjugate distribution are

$$m' = -0.4564$$
$$s'^2(\bar{z}) = 0.137$$
$$n' = 9$$

An estimated prior Bayesian distribution obtained by application of the above algorithm with 1000 samples (a figure used in all subsequent simulations) is shown in Figure 6.

Distance **L** *and Number* **N** *of Inrush Events*

The distance **L** follows an exponential distribution, since the number of inrushes **N** along the axis of area A has been hypothesized to follow a Poisson distribution with mean λ; the conjugate family of both exponential and Poisson distributions are of the gamma-two type. As for yield **q**, a distinction is made between design and subsequent stages.

Design stage: Prior estimation. Prior information may again be obtained by regional estimation, geological analogy, and hydraulic modeling. Here we consider regional estimation.

Figure 7 shows the relationship between FL/A and λ for the Tatabanya coal field by using data from Table 1. Since the mine will be located $y = 0.3$ km from the edge of the karstic aquifer, fissure conditions are represented by a value of $FL/A = 20$ in Figure 5. Returning to Figure 7, a mean $m'(\lambda) = 80$ is predicted. The variance of the predicted value of λ is

$$s'^2(\lambda) = \frac{n'}{n' - 2}(1 - r^2)s^2 \tag{10}$$

where n' is the number of points used for the regression analysis, r is the correlation coefficient, and s^2 is the sample variance; here, $s'^2(\lambda) = 1292$. These two prior parameters specify a conjugate gamma distribution for λ with parameters α' and β' given by

$$m'(\lambda) = \frac{\alpha'}{\beta'} \qquad s'^2(\lambda) = \frac{\alpha'}{\beta'^2} \tag{11}$$

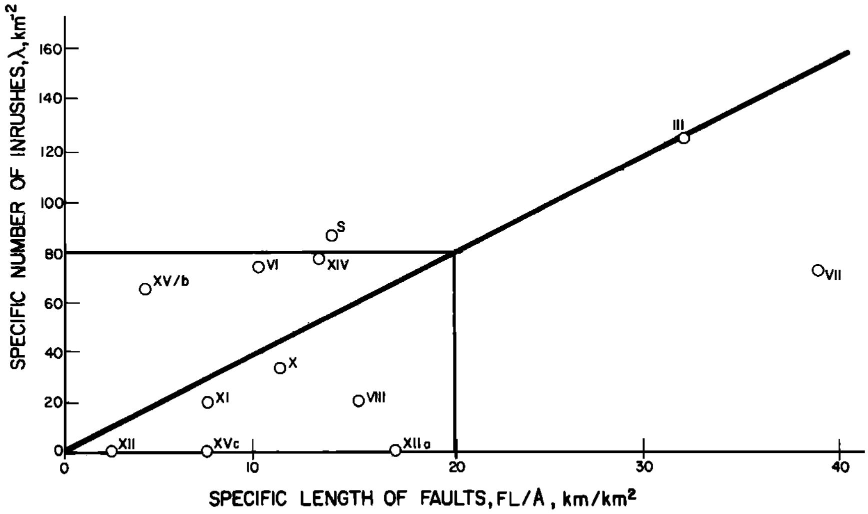

Fig. 7. Relationship between *FL/A* and λ for the Tatabanya coal field [after *Willems,* 1973].

In the case study area, over 500 observed inrush events have been used to construct empirical prior distributions of λ under various hydrogeological conditions. Examples of such fitted empirical distributions of λ are shown in Figure 8. Note the difference between the two coal fields and the effect of protective layer thickness m on λ.

Construction and operation stage: Posterior updating. During the first two years of operation of the mine, $n = 10$ inrushes have occurred over an area $A_d = 0.2$ km². Given prior parameters α' and β' of the distribution of λ, posterior parameters α'' and β'' can be calculated as

$$\alpha'' = \alpha' + n \qquad \beta'' = \beta' + A_d \tag{12}$$

Although an analytic form of the Bayesian distribution of N is available [*Benjamin and Cornell,* 1970] a simulation procedure has been used so that it can be incorporated into the estimation procedure of the derived Bayesian distributions of $\mathbf{q}_{max}$ and **Q**.

A continuous fit to the simulated prior and posterior Bayesian distributions of **N**, with $A = 0.5$ km², is shown in Figure 9.

Maximum Yield of Inrush Events $\mathbf{q}_{max}$

The distribution of $\mathbf{q}_{max}$ has three unknown statistical parameters (equation (1)). Although the distributions of these parameters are known, no analytical procedure is available to derive a closed form Bayesian distribution. A numerical solution would require a triple integration (one for each parameter); further calculations are then necessary for updating the estimates by means of Bayes rule. Here the simulation method seems to provide the only practical approach. Figure 10 shows prior and posterior Bayesian distributions that correspond to the distributions of **q** in Figure 6 and of **N** in Figure 9.

Sum of Inrush Event Yields Q

Figure 11 shows estimated prior and posterior Bayesian distributions of **Q** that correspond to the same distributions of **q** and **N** as shown in Figures 6 and 9, respectively.

DISCUSSION AND CONCLUSIONS

In the engineering design of mines, subways, tunnels that involve soil and rock mechanics, serious problems are caused by lack of knowledge of the physical properties of the system, such as transmissivity or distribution of fissures. Laboratory data, in situ sampling, regional estimation, geological analogy, and engineering judgment may be used to estimate those properties. The stochastic nature of soil properties had made it necessary to estimate stochastic properties of the parameters needed for design. Furthermore, high parameter uncertainty leads to the use of a Bayesian approach. During the engineering design, prior pdf's of design variables are estimated. Subsequently, the construction and operation of the underground works provides observation data on the actual value of those design quantities.

In the karstic aquifer considered, inrush yield and the number of inrush events have been taken as random variables whose pdf may, theoretically at least, be estimated by analytical methods. However, numerical difficulties are such that simulation appears to be the only practical method.

Next, the three basic assumptions which have been made

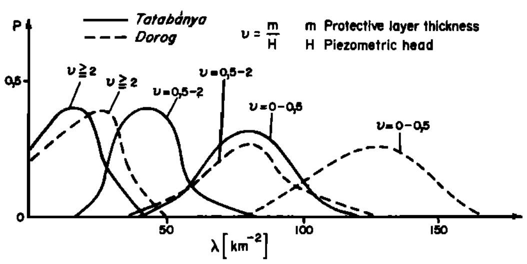

Fig. 8. Fitted empirical distributions of interarrival frequency λ.

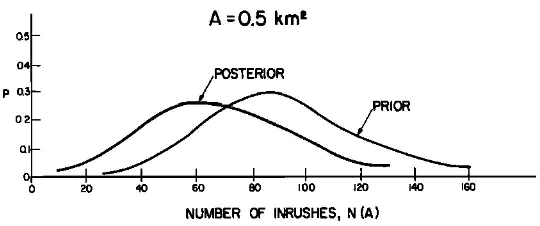

Fig. 9. Simulated prior and posterior distributions of inrush event number N.

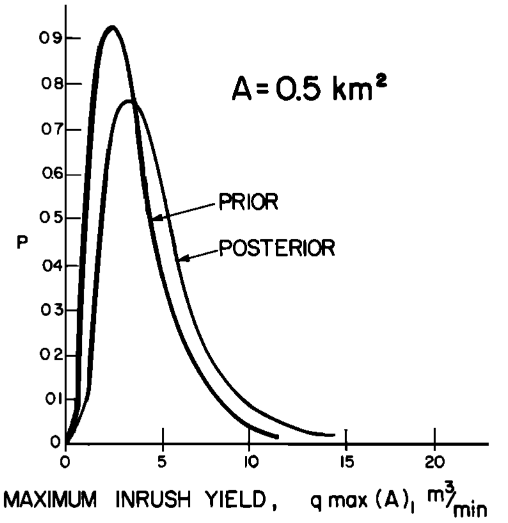

Fig. 10. Simulated prior and posterior distributions of maximum inrush yield $\mathbf{q}_{max}$.

on the distributions of, respectively, **q**, N(*A*), and $\mathbf{q}_{max}$, are discussed in greater detail.

1. The physical reasoning of lognormality for inrush yield **q** [*Bogardi et al.*, 1980] stems from the relationship between the width of fractures **w** and inrush volume **q**. Specifically, it is generally accepted that the width of fractures **w** follows a log normal distribution [*Snow*, 1970]; furthermore, empirical power-type relationships have been established between **w** and **q** [*Schmieder et al.*, 1975], so that it seems reasonable to hypothesize that **q** is also log normally distributed. The validity of this physically based assumption has been checked by analyzing observations taken on more than 500 inrush events. It may be noted that the reason for undertaking the series of inrush event observations was that inrushes kept disturbing mining operations.

2. Inrush locations, that is, the distances between fractures, may be spatially correlated as described, for instance, by *Miller* [1979]. In such a case, methods of geostatistics using variograms [*Journel and Huijbregts*, 1978] can be applied to estimate the stochastic properties of inrush location. However, no Bayesian methodology combined with variogram analysis is known to the authors. And then, under the case study conditions, spatial independence of inrush location may reasonably be assumed. First, there is a physical reason to justify the hypothesis, namely, the volume of water surrounding the coal deposits is extremely large and inrushes may be considered as resulting from a constant piezometric pressure. Second, analysis of the aforementioned observations on more than 500 events offers no basis to reject the hypothesis of spatial independence.

3. The stochastic hydraulic model used herein assumes spatially independent inrush yields; with slight modifications, it may be used to model approximations of hydraulically dependent yields [*Bogardi et al.*, 1980]. Strictly speaking, independent yield volumes occur when the energy in the aquifer is much larger than the total energy loss caused by the cumulative number of inrushes. An exact hydraulic model is available for dependent flows [*Schmieder*, 1976]; the Bayesian analysis in that case is the subject of further investigation.

The results of this paper lead to the following conclusions:

1. Stochastic methods must be used to forecast underground inflow phenomena in a karstic aquifer.
2. Lack of observation in the design stage causes considerble sample uncertainty, which may be accounted for by estimating Bayesian pdf of the design variables.
3. Prior physical and statistical information used in the design stage can be updated with sample information gathered during construction or operation.
4. The design of underground works under karstic water hazards requires the estimation of the pdf of five characteristics of water inrush events.
5. Three of these characteristics (inrush event yield, distance between events, number of events per unit area) are observed, two others (maximum yield, total yield) are derived random variables.
6. Assumptions about the pdf of the observed random variables have been made on a physical basis and are supported by empirical evidence.
7. Simulation seems to provide the only practical approach to estimate the pdf of the derived, hence the observed random variables.

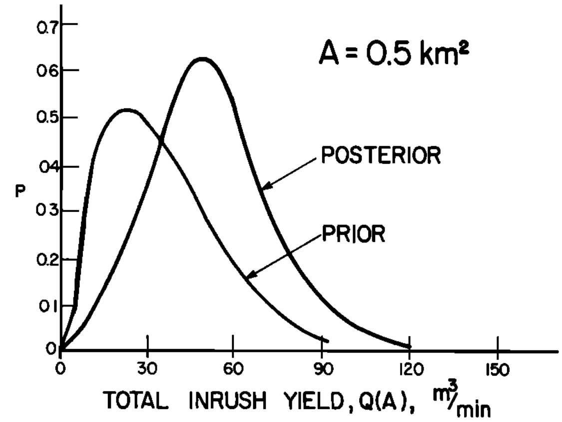

Fig. 11. Simulated prior and posterior distributions of total seasonal inrush **Q**.

Acknowledgments. Research leading to this paper was sponsored in part by funds from the National Science Foundation under grant INT 78-12184 entitled Decision-Making in Natural Resources Management and matching funds from the Central Mining Development Institute, Budapest, Hungary.

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(Received December 2, 1980;
revised January 28, 1982;
accepted February 22, 1982.)